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Mathematics

By the end of Grade 3

1. Students will apply a wide variety of mathematical concepts, processes, and reasoning skills to investigate, evaluate, justify and solve a broad range of problems in various content areas and everyday situations.

   a. Formulate a problem, determine information required to solve the problem, choose methods for obtaining this information, and set limits for acceptable solutions.

   b. Demonstrate that there may be multiple ways to solve a problem and explain why this is so.

   c. Understand that there is no one right way to solve mathematical problems but that different methods (e.g., working backward from a solution, using a similar problem type, identifying a pattern) have different advantages and disadvantages.

   d. Transfer strategies from a prior problem to a new situation.

   e. Use trial and error and the process of elimination to solve problems.

   f. Verify the correctness and reasonableness of simple mathematical results.

2. Students will select and use a wide variety of tools and technology to support and validate mathematical results.

   a. Represent and examine mathematical situations using concrete materials and computers.

   b. Use a four-function or fraction calculator to confirm computations and to explore patterns.

   c. Use a variety of standard tools (e.g., rulers, clocks, measuring tapes, thermometers) and non-standard objects (e.g., counters, sticks, bolts), to measure mathematical and physical objects in the environment.

3. Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems.

   a. Connect physical, verbal, and symbolic representations of whole numbers, fractions and mixed numbers.

   b. Use drawings, diagrams, and models to show the concept of fractions as part of a whole and part of a set.

   c. Explain how numbers are used in various ways, including counting, ordering, representing quantities, measuring, labeling, and indicating location.

   d. Explain how numbers are used in various ways, including counting, ordering, representing quantities, measuring, labeling, and indicating location.

    e.   Apply place-value concepts and numeration to describe, compare, count, order, and group numbers.

    f.   Explain the connections between operations.

    g.   Use concrete objects to count, order, group, and demonstrate one-to-one correspondence with whole numbers beyond 100.

    h.   Identify patterns in number sequences (identify even and odd numbers, count by 2s, 3s, 5s, 10s, and 25s).

    i.   Read, write, and order numbers to 10,000.

4. Students will estimate, compute, and assess reasonableness of solutions.

    a.   Demonstrate proficiency with and memorize addition and subtraction facts through 20 and multiplication facts through 10.

    b.   Add and subtract single- and multi-digit whole numbers with regrouping.

    c.   Apply addition and subtraction in a variety of situations (such as computing perimeter, extending functions).

    d.   Multiply multi-digit whole numbers by single-digit numbers.

    e.   Divide two-digit whole numbers by single-digit numbers.

    f.   Demonstrate the concept of multiplication as repeated addition and arrays; demonstrate the concept of division as repeated subtraction and as sharing.

    g.   Understand and appropriately use the vocabulary of estimation (such as about, near, between).

    h.   Use a variety of mental computational methods, strategies, and estimation skills to find solutions and to determine the reasonableness of calculated answers, including those involving concrete and abstract items and situations, such as time and money.

    i.   Determine the value of a set of host country currency and U.S. currency.

    j.   Read, write, add, and subtract with decimal notation in situations involving money.

5. Students will estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools, and technologies.

    a.   Estimate before measuring to determine the reasonableness of a solution.

    b.   Estimate and measure length, time, temperature, and weight to the nearest unit using customary, metric, and nonstandard measurement.

    c.   Compare and order measurable characteristics (for example, time, temperature, length, weight, capacity, area, perimeter) of different objects on the same dimensions.

    d.   Tell time to the minute with both analog and digital clocks.

    e.   Determine elapsed time to the hour using AM and PM.

  f. Find the perimeter and area of rectangles with direct methods, including using concrete objects as tools.

  g. Recognize the need for a uniform unit of measure.

6. Students will use algebraic methods to represent, analyze, and solve abstract and practical mathematical situations involving patterns and functional relationships.

  a. Recognize, reproduce, extend, create, and describe repeating and increasing patterns and sequences using a variety of materials.

  b. Use tables, graphic organizers, verbal rules, and open sentences to describe patterns and other relationships.

  c. Generate and solve simple functions by identifying and applying addition and subtraction patterns.

  d. Generate, write, and solve open sentences using informal methods (such as using manipulatives, drawing, or acting out the solution).

  e. Use concrete objects and symbols to model the concepts of variables, expressions, equations, and inequalities (for example, find the missing number, symbol, or operation sign).

  f. Identify and describe numeric patterns and make predictions based on them (e.g., 1 bicycle = 2 wheels, so 6 bicycles = how many wheels).

7. Students will use spatial reasoning and apply the properties and relationships of geometric figures to represent, investigate, analyze, and solve problems.

  a. Use comparative directional and positional words (such as above, inside, left, horizontal, middle).

  b. Describe, name, and label related geometric two- and three-dimensional shapes (such as circle and sphere, square and cube, triangle and pyramid, rectangle and prism).

  c. Draw two-dimensional geometric shapes and construct rectangles, squares, and triangles using tools (such as geoboards, grid paper, ruler, compass), including representation of side, top, and bottom views of the object.

  d. Construct three-dimensional geometric shapes, including boxes and triangular prisms.

  e. Identify and describe geometric figures in the environment.

  f. Identify and create examples of line symmetry.

  g. Order simple geometric figures by size.

  h. Estimate and determine the perimeter and area of geometric figures using manipulatives; demonstrate conservation of area.

  i. Describe, identify, and model slides, flips, and turns with geometric figures.

8. Students will use statistical methods to question, collect, organize, analyze, and represent data in order to make decisions and predictions.

   a. Pose a question and collect data by observing, measuring, surveying, and counting

   b. Construct, read, interpret, and label graphs, including pictographs, simple bar and line graphs, and pie charts.

   c. Interpret data by looking for patterns and relationships, determining range, considering cause and effect, then drawing conclusions and answering related questions.

9. Students will understand and apply basic concepts of probability.

   a. Predict and measure the outcome of events, and explain why the results of an experiment may not match predicted outcomes.

   b. Use concepts of certainty, fairness, and chance to discuss the probability of actual events.

By the end of Grade 5

1. Students will apply a wide variety of mathematical concepts, processes, and reasoning skills to investigate, evaluate, justify and solve a broad range of problems in various content areas and everyday situations.

   a. Know how to select and use mathematical tools and methods (such as manipulatives, mental math, calculator, computer, and paper-and-pencil techniques) as a part of the problem-solving process.

   b. Develop and apply a variety of problem-solving strategies (for example, make an organized list, guess-and-check) and justify choice of strategies.

   c. Interpret results in the context of the problem being solved (for example, when determining the number of buses necessary to transport students, the remainder must be rounded up).

   d. Differentiate between relevant and irrelevant information.

   e. Understand how to break a complex problem into simpler parts.

2. Students will select and use a wide variety of tools and technology to support and validate mathematical results.

   a. Use calculators or software to verify estimations and in problem-solving situations.

   b. Use technology such as spreadsheets, cameras, science probe, or calculators to gather, analyze, and display mathematical data and information.

3. Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems.

   a. Model and connect physical, verbal, and symbolic representations of fractions,decimals, percentages, whole numbers, and mixed numbers.

   b. Order fractions, decimals, and whole numbers using physical, verbal, and symbolic representations.

   c. Recognize the relationship among fractions, decimals, and percentages.

d.   Use concepts of negative numbers in concrete situations (such as on a number line, with temperature).

e.   Identify and describe different uses for the same numerical representation (for

f.   Use, model, and identify place value and describe its relationship to magnitude. example, 1/4 can represent a fraction, a division problem, or a ratio) and different representations for the same number (for example, 2,343 is the same as 2,000 + 300 + 40 + 3; and 1 equals 16/16).

g   Demonstrate that mathematical operations can represent a variety of problem situations (for example, multiplication can represent repeated addition and a model for finding area).

h.   Explain the relative effect of operations with fractions and decimals (for example, what happens to 10 when you divide by 1/2 or multiply by .75).

i.   Explain, derive, compare, and use properties of operations and relationships among operations.

i.   Explain and apply number theory concepts (such as primes, multiples, and composites).

j.   Read, write, and order numbers to a million.

4.   Students will estimate, compute, and assess reasonableness of solutions.

a.   Demonstrate proficiency with and memorize multiplication and division facts through 12.

b.   Select and use the most efficient computational methods, choosing among concrete materials, paper and pencil, estimation, mental computation, and calculators.

c.   Create and solve practical problems involving addition, subtraction, multiplication and division.

d.   Addition, subtraction, multiplication, and division of whole numbers, fractions, and mixed numbers.

e.   Develop, analyze, and compare algorithms for computing with fractions, decimals, percents, and integers and compute with them efficiently and accurately, including in multi-step problems that require application of order of operations.

f.   Know and convert among fractions, decimals, and percents for 1/10, 1/5, 1/4, 1/2, and 3/4.

g.   Apply beginning number theory including identifying and using multiples, factors, divisibility, properties of identity (zero and one), and prime and composite numbers.

h.   Apply, explain, and assess the appropriateness of a variety of estimation strategies (such as rounding to compatible numbers).

    i. Use various forms of estimation, including rounding, to determine the reasonableness of calculated answers; determine if an estimate is too high or too low.

    j. Use a variety of strategies to make change and solve problems using U.S. and host country's currency, and to convert between host country and U.S. currency.

5. Students will estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools, and technologies.

    a. Select and use appropriate instruments and customary and metric units for measuring quantities, including perimeter, volume, area, weight, time, and temperature, with specified accuracy; match tools with the attribute they measure (for example, rulers measure length, thermometers measure temperature).

    b. Understand and apply formulas for finding perimeter, volume of simple solids (excluding cylinders), and area.

    c. Add and subtract measurements (e.g., 12 m. – 6.2 m.).

    d. Identify and use equivalent measurements as required by the situation (for example, 60 minutes = 1 hour, 7 days = 1 week).

    e. Identify the approximate size of basic standard units of measurement and the relationship between them (for example, there are 100 centimeters in a meter).

    f. Solve calendar problems involving days, weeks, months, and years.

    g. Determine and compare elapsed time using AM and PM and a 24-hour clock.

6. Students will use algebraic methods to represent, analyze, and solve abstract and practical mathematical situations involving patterns and functional relationships.

    a. Use patterns and their extensions to make predictions and solve problems.

    b. Use rules and variables to describe patterns, functions, and other relationships and to solve equations.

    c. Generate and solve simple functions by identifying and applying multiplication and division patterns.

    d. Find solutions to inequalities from a given replacement set.

    e. Solve simple equations using methods such as inverse operations, mental math, and guess-and-check.

    f. Use concrete objects and combinations of symbols and numbers to create expressions that model mathematical situations.

    g. Understand the basic characteristics of a 2-dimensional coordinate system.

7. Students will use spatial reasoning and apply the properties and relationships of geometric figures to represent, investigate, analyze, and solve problems.

a. Locate and describe objects in terms of their position with and without compass directions; identify coordinates for a given point or locate points of given coordinates on a single quadrant grid.

b. Compare, contrast, and describe plane and solid figures and shapes using their attributes (such as number of sides, parallel or perpendicular sides, number of vertices, classification of angles).

c. Sketch and identify line segments, midpoint, intersections, and parallel and perpendicular lines.

d. Identify, draw, and measure, using a protractor, right, obtuse, and acute angles and their parts, including rays, points, and vertices.

e. Identify and model geometric figures that are congruent, similar, or symmetrical or some combination of these properties.

f. Identify the diameter, radius, chord, and circumference of a circle.

g. Determine area and perimeter, finding both using a variety of methods.

h. Analyze and model transformations of geometric figures and rotations of line segments, describing the motions as slides, flips, or rotations.

8. Students will use statistical methods to question, collect, organize, analyze, and represent data in order to make decisions and predictions.

a. Solve problems that involve systematically collecting, organizing, and analyzing data.

b. Discuss the appropriateness of different types of data displays, and use a variety of displays (such as tables, histograms, graphs).

c. Interpret data, using the arithmetic mean, median, mode, range, and make convincing arguments based on data analysis and previous experiences.

d. Find all possible outcomes of a simple experiment using straightforward methods (such as organized lists, tree diagrams).

9. Students will understand and apply basic concepts of probability.

a. Make predictions based on intuitive, experimental, and theoretical probabilities.

b. Conduct simple probability experiments using concrete materials (e.g. tossing one or more coins, spinning a spinner of even or uneven divisions, drawing objects from a container with and without replacement) and represent the results using fractions and probability.

By the end of Grade 8

1. Students will apply a wide variety of mathematical concepts, processes, and reasoning skills to investigate, evaluate, justify and solve a broad range of problems in various content areas and everyday situations.

a. Pose, explore, and solve a variety of problems, including those that are non-routine or have a variety of possible strategies or solutions or both, in order to build new mathematical knowledge.

b. Develop, modify, and apply an increasing variety of problem-solving techniques to solve problems (for example, working backward, information organizers, or solving a similar but simpler problem).

c. Try various problem-solving approaches before selecting and using a strategy and reflect on different strategies used when a task is complete.

2. Students will select and use a wide variety of tools and technology to support and validate mathematical results.

a. Use a variety of technologies, including computers, scientific calculators, graphing calculators, science probes, and digital cameras to evaluate and validate problem solutions.

b. Recognize situations when calculator use is not appropriate (for example, when solving a simple quadratic equation which could be factored) or when it yields misleading results (for example, when a non-linear curve appears linear).

3. Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems.

a. Demonstrate a conceptual understanding of irrational numbers and be able to solve problems involving rational numbers (e.g., area of circles, working with radicals).

b. Describe how percent, ratio, and proportion apply to mathematical situations (such as rate, similar triangles).

c. Recognize and apply multiple representations of rational and irrational numbers, exponents, absolute values, and scientific notation; compare these numbers accurately, find their approximate locations on a number line, and choose appropriate forms of these numbers.

d. Demonstrate an understanding of numbers that represent large and small values, including the use of benchmarks to comprehend their magnitude; and recognize, understand, and appropriately use various representations for large numbers.

e. Represent and explain the effect of operations on positive and negative numbers.

f. Add, subtract, multiply, and divide fractions, decimals, percents, integers, and nonnegative whole number exponents.

g. Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.

h. Use factors, multiples, and prime factorization to solve problems.

i. Recognize and use the associative and commutative properties of addition and multiplication, and the distributive property of multiplication over addition to simplify computations with rational numbers.

j. Model and connect physical, verbal, and symbolic representations of real numbers by hand and/or with a graphing calculator.

4. Students will estimate, compute, and assess reasonableness of solutions.

a. Solve problems using rates and understand rate as a unit of measure.

b. Use algorithms for computing with fractions, decimals, percents, and integers and compute them efficiently and accurately with and without a calculator.

c. Use multi-step computational procedures with rational and irrational numbers.

d. Estimate the value of irrational numbers.

e. Develop, analyze, and explain methods to solve problems involving proportions and percents (such as scaling, finding equivalent ratios).

f. Compute circumference, area, surface area, and volume of geometric figures; find missing dimensions of right triangles using the Pythagorean theorem.

g. Estimate the value of tips, discounts, and taxes using host country and U.S. currency.

h. Explain and apply the rules of divisibility, square numbers, prime factorization, and the properties of zero with the order of operations.

i. Determine what a reasonable degree of accuracy would be in particular situations (e.g., great precision is required in scientific experiments, but much less is required in estimating a grocery bill).

5. Students will estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools, and technologies.

a. Estimate and measure angles and use formulas to find perimeter, area, and circumference of plane figures and the volume and surface area of prisms, pyramids, and cylinders to a specified degree of accuracy.

b. Select and use appropriate units and tools to measure length, area, volume, angle, and weight to appropriate levels of precision.

c. Convert measurements within and between monetary systems and within and between metric and customary systems and demonstrate an understanding of the relationship between units in metric and customary systems.

d. Select and apply indirect methods of measurement including formulae, geometric ideas and relationships (such as congruence, similarity and the Pythagorean theorem) and algorithms for mathematical relationships (such as scale, proportions, rate).

e. Select and use a variety of methods and tools to construct and compare plane figures of given measures.

f. Apply information about time zones and elapsed time to solve problems.

6. Students will use algebraic methods to represent, analyze, and solve abstract and practical mathematical situations involving patterns and functional relationships.

   a. Analyze, create, and generalize numeric and visual patterns.

   b. Describe relationships between symbolic expressions and graphs on the coordinate plane, with particular attention to the horizontal and vertical intercepts, points of intersection, and slope (for linear relations).

   c. Analyze functional relationships to explain how a change in one quantity results in a change in another (for example, the relationship among length, area, and volume).

   d. Find the value of a variable by evaluating formulas and algebraic expressions for given values (for example, if an object has a length of 4 and an area of 28, what is the object's width?).

   e. Rewrite formulas in terms of the missing variable (for example, "if an object has a length of 4 and an area of 28, what is the object's width?" can be expressed as 4W=28 or as W=28/4).

   f. Create expressions, equations, and inequalities to represent problem situations and to solve problems involving linear relationships.

   g. Demonstrate fluency in generating equivalent expressions for simple algebraic expressions and in solving linear equations and inequalities.

   h. Demonstrate a basic understanding of rate of change, including connections between slope of a line and constant rate of change and their meaning in context.

7. Students will use spatial reasoning and apply the properties and relationships of geometric figures to represent, investigate, analyze, and solve problems.

   a. Describe the relationship between an equation and its graph.

   b. Use coordinate geometry to represent and interpret relationships defined by equations and formulas (for example, distance, mid-point), translating among ordered pairs, graphs, and equations.

   c. Model, classify, compare, and sketch a variety of two- and three-dimensional regular and irregular figures.

   d. Apply properties of equality and proportionality to solve problems involving congruent or similar shapes.

   e. Describe and apply geometric properties and relationships (such as congruence, perpendicularity).

   f. Describe and apply a variety of strategies for determining circumference, perimeter, area, surface area, angle measure, and volume.

   g. Explain and apply the Pythagorean theorem.

   h. Draw and describe the results of transformations, including translations, rotations, reflections, and dilations (shrinking or enlarging), using proper notation.

8 . Students will use statistical methods to question, collect, organize, analyze, and represent data in order to make decisions and predictions.

   a.   Formulate and solve problems by collecting, organizing, analyzing (including comparing and contrasting), and displaying data in a variety of ways (including stem and leaf plots, histograms, whisker plots, surveys, circle graphs) by hand and with technology.

   b.   Find, describe, and interpret the arithmetic mean, median, and mode as measures of the center of a data set; select the appropriate measure in particular situations to most accurately and fairly represent the data.

   c.   Describe ways to define a sample group; analyze a sample to make inferences about a population.

   d.   Design and conduct a simulation to study a problem and communicate the results.

9.   Students will understand and apply basic concepts of probability.

   a.   Determine theoretical probability using a variety of methods, including creating a sample space; compare theoretical expectations to experimental results.

   b.   Design, conduct, and analyze the results of probability experiments.

By the end of Grade 12

1.   Students will apply a wide variety of mathematical concepts, processes, and reasoning skills to investigate, evaluate, justify and solve a broad range of problems in various content areas and everyday situations.

   a.   Explore the validity and efficiency of various problem-posing and problem-solving strategies; develop alternative strategies and generalizations as needed.

   b.   Monitor progress toward solutions.

   c.   Generalize strategies and reflect on their proficiency and merit.

2.   Students will select and use a wide variety of tools and technology to support and validate mathematical results.

   a.   Use graphing calculators and computer software effectively and efficiently to define and solve various types of problems.

3.   Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems.

   a.   Explain the effect of operations on measurements (for example, the imprecise nature of measurement is amplified with multiplication).

   b.   Understands the concept of infinity.

4.   Students will estimate, compute, and assess reasonableness of solutions.

   a.   Manipulate algebraic procedures with real and complex numbers.

    b.   Apply factorials, exponents, and matrices to solve practical problems.

    c.   Compute permutations and combinations.

    d.   Assess the error resulting from estimation and rounding, using both customary and metric units.

    e.   Estimate algebraic solutions.

    f.   Determine when to use exact value solutions and distinguish between exact value and approximate values.

5. Students will estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools, and technologies.

    a.   Incorporate units into all aspects of measurement problems and determine the appropriateness of a solution based upon dimensional analysis.

    b.   Explain the relationship among error, precision, and accuracy in measurement, including the compounding of errors in calculations.

    c.   Evaluate the accuracy and precision of measurements resulting from the measuring tools and methods chosen.

    d.   Apply indirect methods, such as ratios and trigonometry, to find missing dimensions.

    e.   Interpret various international measurement systems (such as the Richter Scale, decibels) to describe phenomena and solve problems.

6. Students will use algebraic methods to represent, analyze, and solve abstract and practical mathematical situations involving patterns and functional relationships.

    a.   Define functions and their properties and find the inverse of a function; understand the relationship between a function and its inverse.

    b.   Create and solve linear and quadratic equations and inequalities.

    c.   Add, subtract, multiply, divide, and simplify rational and irrational expressions; add, subtract, multiply and divide polynomials.

    d.   Identify, graph, and describe the graphs of basic families of functions including linear, absolute value, quadratic, exponential, and trigonometric functions and explain why a variety of phenomena can be modeled by the same type of function.

    e.   Solve systems of equations and inequalities.

    f.   Formulate expressions, equations, inequalities, systems of equations, systems of inequalities, and matrices to model problem situations.

    g.   Use matrices to organize and manipulate data, including matrix addition, subtraction, multiplication, and scalar multiplication.

7. Students will use spatial reasoning and apply the properties and relationships of geometric figures to represent, investigate, analyze, and solve problems.

a. Be familiar with the graphs of the following equations and be able to apply them to problem solving: $y = \sin x$; $y = \cos x$; $y = \tan x$.

b. Use coordinate geometry to graph linear and quadratic equations, determine slopes of lines, identify parallel and perpendicular lines, and find possible solutions to sets of equations.

c. Create two-dimensional representations of three-dimensional objects (e.g., draw a basic cube).

d. Construct geometric models, transformations, and scale drawings using a variety of methods and tools (such as paper folding or protractor).

e. Identify congruent and similar figures; apply this information to solve problems

f. Use basic trigonometric ratios and trigonometric laws (triangle trigonometry) to solve problems involving indirect measurement.

g. Use vector methods, matrices, and transformations

8. Students will use statistical methods to question, collect, organize, analyze, and represent data in order to make decisions and predictions.

   a. Describe the attributes of several common distributions (e.g., normal, uniform,Poisson, exponential), indicating data sets that would be expected to follow each type of distribution

   b. Determine regression equations to model and draw inferences from data; summarize and interpret single-variable data by choosing measures of central tendency and dispersion.

   c. Analyze the validity of statistical conclusions and the use, misuse, and abuse of data caused by choices of scale, inappropriate choices of central tendency, incorrect curve fitting, or inappropriate use of control groups.

   d. Pose questions; collect, organize, and represent data to answer those questions.

9. Students will understand and apply basic concepts of probability.

   a. Describe the normal curve in general terms and use its properties to answer questions about sets of data.

   b. Find the probability of simple events, compound events, and independent events using a variety of methods including the fundamental counting principle.

Communication

Grade 1

o Students will:

   ▪ 1. Express mathematical ideas coherently and clearly to peers, teachers, and others (e.g., with verbal ideas, models or manipulatives, pictures, or symbols).

- ▪ 2. Extend mathematical knowledge by considering the thinking and strategies of others (e.g., agree or disagree, rephrase another student's explanation, analyze another student's explanation).

- ▪ 3. Relate manipulatives, pictures, diagrams, and symbols to mathematical ideas.

- ▪ 4. Represent, discuss, write, and read mathematical ideas and concepts. Start by relating everyday language to mathematical language and symbols and progress toward the use of appropriate terminology (e.g., "add more" becomes "plus", "balance the equation" becomes "solve the equation").

Grade 2

- o Students will:

  - ▪ 1. Express mathematical ideas coherently and clearly to peers, teachers, and others (e.g., with verbal ideas, models or manipulatives, pictures, or symbols).

  - ▪ 2. Extend mathematical knowledge by considering the thinking and strategies of others (e.g., agree or disagree, rephrase another student's explanation, analyze another student's explanation).

  - ▪ 3. Relate manipulatives, pictures, diagrams, and symbols to mathematical ideas.

  - ▪ 4. Represent, discuss, write, and read mathematical ideas and concepts. Start by relating everyday language to mathematical language and symbols and progress toward the use of appropriate terminology (e.g., "add more" becomes "plus", "repeated addition" becomes "multiplication", "fair share" becomes "divide", "balance the equation" becomes "solve the equation").

Grade 3

- o Students will:

  - ▪ 1. Express mathematical ideas coherently and clearly to peers, teachers, and others (e.g., with verbal ideas, models or manipulatives, pictures, or symbols).

  - ▪ 2. Extend mathematical knowledge by considering the thinking and strategies of others (e.g., agree or disagree, rephrase another student's explanation, analyze another student's explanation).

  - ▪ 3. Relate manipulatives, pictures, diagrams, and symbols to mathematical ideas.

  - ▪ 4. Represent, discuss, write, and read mathematical ideas and concepts. Start by relating everyday language to mathematical language and symbols and progress toward the use of appropriate terminology (e.g., "add more" becomes "plus", "repeated addition" becomes "multiplication", "fair share" becomes "divide", "balance the equation" becomes "solve the equation").

Grade 4

- o Students will:
    - 1. Relate manipulatives, pictures, diagrams, and symbols to mathematical ideas.
    - 2. Represent, discuss, write, and read mathematical ideas and concepts. Start by relating everyday language to mathematical language and symbols and progress toward the use of appropriate terminology (e.g., "add more" becomes "plus", "repeated addition" becomes "multiplication", "fair share" becomes "divide", "balance the equation" becomes "solve the equation").

Grade 5

- o Students will:
    - 1. Express mathematical ideas coherently and clearly to peers, teachers, and others (e.g., with verbal ideas, models or manipulatives, pictures, or symbols).
    - 2. Extend mathematical knowledge by considering the thinking and strategies of others (e.g., agree or disagree, rephrase another student's explanation, analyze another student's explanation).
    - 3. Relate manipulatives, pictures, diagrams, and symbols to mathematical ideas.
    - 4. Represent, discuss, write, and read mathematical ideas and concepts. Start by relating everyday language to mathematical language and symbols and progress toward the use of appropriate terminology (e.g., "add more" becomes "plus", "repeated addition" becomes "multiplication", "fair share" becomes "divide", "balance the equation" becomes "solve the equation").

Connections

Grade 1

- o Students will:
    - 1. Relate various concrete and pictorial models of concepts and procedures to one another (e.g., use two colors of cubes to represent addition facts for the number 5, count by 2s, 5s, 10s)
    - 2. Link concepts to procedures and eventually to symbolic notation (e.g., represent actions like snap, clap, clap with symbols A B B, divide a candy bar into 3 equal pieces that represent one piece as 1/3).
    - 3. Recognize relationships among different topics within mathematics (e.g., the length of an object can be represented by a number)
    - 4. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world (e.g., use a timeline to sequence events,

use symmetry in art work, explore fractions in quilt designs and to describe pizza slices).

Grade 2

- o Students will:
    - 1. Relate various concrete and pictorial models of concepts and procedures to one another (e.g., use two colors of cubes to represent addition facts for the number 5, relate patterns on a hundreds chart to multiples, use base-10 blocks to represent decimals).
    - 2. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world (e.g., use a timeline to sequence events, use symmetry in art work, explore fractions in quilt designs and to describe pizza slices).

Grade 3

- o Students will:
    - 1. Relate various concrete and pictorial models of concepts and procedures to one another (e.g., use two colors of cubes to represent addition facts for the number 5, relate patterns on a hundreds chart to multiples, use base-10 blocks to represent decimals).
    - 2. Recognize relationships among different topics within mathematics (e.g., the length of an object can be represented by a number, multiplication facts can be modeled with geometric arrays, ½ can be written as .5 and 50%).
    - 3. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world (e.g., use a timeline to sequence events, use symmetry in art work, explore fractions in quilt designs and to describe pizza slices).

Grade 4

- o Students will:
    - 1. Relate various concrete and pictorial models of concepts and procedures to one another (e.g., use two colors of cubes to represent addition facts for the number 5, relate patterns on a hundreds chart to multiples, use base-10 blocks to represent decimals).
    - 2. Link concepts (e.g., fractions, multiplication, division) to procedures and eventually to symbolic notation (e.g., demonstrate 3 • 4 with a geometric array, divide a candy bar into 3 equal pieces that represent one piece as 1/3).
    - 3. Recognize relationships among different topics within mathematics (e.g., the length of an object can be represented by a number, multiplication facts can be modeled with geometric arrays, ½ can be written as .5 and 50%).

- 4. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world (e.g., use a timeline to sequence events, use symmetry in art work, explore fractions in quilt designs and to describe pizza slices).

Grade 5

- o Students will:

    - 1. Relate various concrete and pictorial models of concepts and procedures to one another (e.g., use two colors of cubes to represent addition facts for the number 5, relate patterns on a hundreds chart to multiples, use base-10 blocks to represent decimals).

    - 2. Link concepts to procedures and eventually to symbolic notation (e.g., represent actions like snap, clap, clap with symbols A B B, demonstrate 3 • 4 with a geometric array, divide a candy bar into 3 equal pieces that represent one piece as).

    - 3. Recognize relationships among different topics within mathematics (e.g., the length of an object can be represented by a number, multiplication facts can be modeled with geometric arrays, can be written as .5 and 50%).

    - 4. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world (e.g., use a timeline to sequence events, use symmetry in art work, explore fractions in quilt designs and to describe pizza slices).

Data Analysis
Pre-Kindergarten

- o Students will:

    - 1. Develop growing abilities to collect, describe, and record information through a variety of means, including discussion, drawings, maps, charts and graphs.

    - 2. Describe similarities and differences between objects

Kindergarten

- o Students will:

    - 1. Collect and analyze information about objects and events in the environment.

    - 2. Create and verbally explain a data display or graph (e.g., real object graph, pictorial graphs)

Grade 1

- o Students will:
  - 1. Organize, describe, and display data using concrete objects, pictures, grids, and numbers.
  - 2. Formulate and solve problems that involve collecting and analyzing data common to children's lives (e.g., color of shoes, numbers of pets, favorite foods).

Grade 2

- o Students will:
  - 1. Collect, sort, organize, and display data in charts, bar graphs, and tables (e.g., collect data on teeth lost and display results in a chart).
  - 2. Summarize and interpret data in charts, bar graphs, and tables.
  - 3. Make predictions and estimates to describe data (e.g., predict what data on teeth lost might look like for younger children and/or older children).

Grade 3

- o Students will:
  - 1. Pose questions, collect, record, and interpret data to help answer questions (e.g., Which was the most popular booth at our carnival?).
  - 2. Read graphs and charts; identify the main idea, draw conclusions, make predictions based on the data (e.g., predict how many children will bring their lunch based on a menu).
  - 3. Construct a bar graph or pictograph with labels and a title from a set of data.

Grade 4

- o Students will:
  - 1. Examine data displays such as tallies, tables, charts and graphs and use the observations to pose and answer questions (e.g., choose a table in social studies of population data and write problems).
  - 2. Collect, organize and record data in tables and graphs (e.g., bar, pictograph, line plots).
  - 3. Investigate and record probabilities by experimenting with devices that generate random outcomes (e.g., coins, number cubes, spinners).

Grade 5

- o Students will:
    - 1. Analyze data to create and interpret tables and graphs.
    - 2. Justify the selection of the type of table or graph (e.g., a line graph may be more appropriate than a bar graph when displaying the height of a person over time).
    - 3. Compare and translate between displays of data (e.g., multiple sets of data on the same graph, Venn diagrams, a combination of diagrams, charts, tables, graphs).
    - 4. Formulate questions, design investigations, consider samples, and collect, organize, and analyze data using observation, measurement, surveys, or experiments (e.g., how far can 5th graders throw a softball based on where it first hits the ground?).
    - 5. Determine the range (spread) and the mean (average) of a set of data.
    - 6. Determine the probability of events occurring in familiar contexts or experiments and express probabilities as fractions (e.g., find the fractional probability of an event given a biased spinner).
    - 7. List permutations and combinations of up to five items.

Geometry and Measurement

Grade 1

- o Students will:
    - 1. Sort and identify congruent shapes.
    - 2. Compare objects by size and quantity (e.g., more than, less than, equal to).
    - 3. Identify two-dimensional geometric shapes and objects in everyday situations (e.g., the face of a round clock is a circle, a desktop is a rectangle).
    - 4. Use language to describe relationships of objects in space (e.g., above, below, behind, between).
    - 5. Tell time on digital and analog clocks to the hour and half-hour.
    - 6. Identify and name the value of local currency
    - 7. Use the calendar to develop the concepts of days, weeks, and months.
    - 8. Identify solid figures and sort by faces and vertices.

Grade 2

- o Students will:

- 1. Sort and classify symmetric and congruent figures.

- 2. Identify two-dimensional geometric shapes in everyday situations (e.g., a stop sign is an octagon).

- 3. Measure objects with nonstandard and standard units (e.g., use a human foot [nonstandard] then a ruler [standard] to measure length to the nearest inch).

- 4. Select and use appropriate units of measurement in problem solving and everyday situations.

- 5. Tell time on digital and analog clocks to the quarter-hour.

- 6. Identify and count money; connect coins and bills with place value.

Grade 3

- o Students will:

  - 1. Describe and compare two- and three-dimensional shapes (e.g., count the edges and faces of a cube, combine or divide basic shapes to form new shapes, identify and draw congruent shapes).

  - 2. Identify locations on a grid with ordered pairs (e.g., give the location of a ship on a grid by selecting D, 1).

  - 3. Solve problems with customary units involving length using half-inch and quarter-inch measurements and weight using pound and ounce.

  - 4. Solve problems with metric units involving length using meter and centimeter and mass using gram and kilogram.

  - 5. Use manipulatives to develop the concept of perimeter and area (e.g., cover a shape with pattern blocks to find area).

  - 6. Develop and use strategies to estimate measurements (e.g., use parts of the body as benchmarks for measuring length).

  - 7. Tell time on digital and analog clocks to 5 minutes and use information to solve problems involving time and temperature (e.g., read a thermometer).

Grade 4

- o Students will:

  - 1. Identify, draw, and construct models of intersecting, parallel, and perpendicular lines (e.g., use spaghetti, straws, toothpicks).

  - 2. Identify and compare angles equal to, less than, or greater than 90 degrees (e.g., use right angles to determine the approximate size of other angles; make a variety of angles using flexible straws and compare).

  - 3. Identify basic characteristics of the rectangular coordinate system and find the distance between horizontal and vertical lines of a rectangular coordinate system (e.g., the x-axis is the horizontal axis).

- ▪ 4. Describe the effects on two- and three-dimensional objects when they slide (translate), flip (reflect), and turn (rotate) (e.g., tessellations).

- ▪ 5. Establish benchmarks for customary and metric units and estimate the measures of a variety of objects and compare units (e.g., mass: the mass of a raisin is about 1 gram, length: the width of a finger is about 1 centimeter).

- ▪ 6. Select appropriate customary and metric units of measure to solve application problems involving length, weight, mass, and volume.

- ▪ 7. Solve application problems involving money, time and temperature (e.g., elapsed time).

Grade 5

- o Students will:

  - ▪ 1. Identify and describe the basic properties of figures (e.g., two or three-dimensionality, symmetry, number of faces, types of angles).

  - ▪ 2. Find the perimeter of simple polygons and area of a rectangle (e.g., use 1-inch tiles to build rectangles of different perimeters and areas).

  - ▪ 3. Use nonstandard units (beans, rice, candies) and standard units (centimeter cubes, 1-inch cubes) to find the volume of rectangular solids and estimate the volume of other solids.

  - ▪ 4. Use the appropriate units and tools to estimate and measure temperature, distance, length, weight, and angles.

  - ▪ 5. Convert basic measurements of volume, weight and distance within the same system for metric and customary units (e.g., inches to feet, hours to minutes, centimeters to meters).

Measurement

Pre-Kindergarten

- o Students will:

  - ▪ 1. Measure objects using non-standard units of measurement (e.g., pencil, paper clip, block)

  - ▪ 2. Compare objects according to observable attributes (e.g., long, longer, longest)

  - ▪ 3. Develop an awareness of simple time concepts within his/her daily life (e.g., yesterday, tomorrow)

  - ▪ 4. Sequence events in the context of daily activities

Kindergarten

- o Students will:

- 1. Measure objects using nonstandard units of measurement (e.g., pencil, paper clip, block)

- 2. Compare objects according to observable attributes (e.g., long, longer, longest, short, shorter, shortest, big, bigger, biggest, small , medium and large)

- 3. Compare and order objects in graduated order (e.g., shortest to tallest, thinnest to thickest)

- 4. Identify the appropriate instrument used to measure length (ruler), weight (scale), time (clock: digital/analog, calendar: day, month, year, season), and temperature (thermometer)

- 5. Identify days of the week and months of the year

Number Operations
Grade 1

- o Students will:

  - 1. Use models to construct addition and subtraction facts through 18 (e.g., counters, cubes).

  - 2. Perform addition by joining sets of objects and subtraction by separating and by comparing sets of objects.

  - 3. Demonstrate fluency with basic addition and subtraction facts (i.e., memorize and apply addition and subtraction facts) through 18. Compute efficiently and accurately with single-digit numbers, double digit with no carrying.

  - 4. Recognize and apply the commutative and identity properties of addition using models and manipulatives to develop computational skills (e.g., 2 + 4 = 4 + 2, 3 + 0 = 3).

  - 5. Write addition and subtraction number sentences for problem-solving situations.

  - 6. Acquire strategies for making computations (e.g., use estimation, number sense to judge reasonableness, counting on).

Grade 2

- o Students will:

  - 1. Demonstrate fluency with basic addition and subtraction facts (i.e., memorize and apply addition and subtraction facts) and fact families to 18.

  - 2. Solve two-digit addition and subtraction problems with and without regrouping using a variety of techniques (e.g., concrete, paper and pencil, mental math).

- 3. Develop operation sense by applying the associative property of addition (e.g., 3 + (2 +1) = (3 + 2) + 1).

- 4. Describe the relationship between addition and subtraction (e.g., addition and subtraction are inverse operations).

- 5. Use mental strategies (or decomposition strategies) for addition and subtraction (e.g., make a group of 10 objects and 2 objects from a group of 7 objects and 5 objects).

- 6. Complete addition number sentences with a missing addend and use to solve everyday problems.

Grade 3

o Students will:

- 1. Estimate, find the sum and difference, with and without regrouping, of 3- and 4-digit numbers to solve application problems.

- 2. Demonstrate fluency with basic multiplication facts and fact families (i.e., memorize and apply multiplication facts).

- 3. Develop multiplication algorithms (e.g., use physical materials to show 4 groups of 3 objects, show multiplication as repeated addition).

- 4. Estimate the product of 2-digit numbers by rounding to the nearest multiple of 10 to solve application problems.

- 5. Recognize and apply the commutative and identity properties of multiplication using models and manipulatives to develop computational skills (e.g., 3 • 5 = 5 • 3, 7 • 1 = 7).

- 6. Solve problems involving money that require addition and subtraction.

Grade 4

o Students will:

- 1. Estimate and find the product of 2- and 3-digit numbers to solve application problems.

- 2. Demonstrate fluency with basic division facts and fact families (i.e., memorize and apply division facts).

- 3. Develop division algorithms (e.g., use physical materials to show 12 objects arranged in 3 groups, show division as repeated subtraction and as the inverse of multiplication).

- 4. Estimate and find the quotient (with and without remainders) with a 1-digit divisor and a 2- or 3-digit dividend to solve application problems.

- 5. Apply a variety of estimation and mental math techniques to simplify computations (e.g., use rounding to estimate 82 - 58 is about 80 - 60 or 20, use 30 • 5 to find the product of 300 • 5).

- 6. Develop operation sense by applying the associative property of multiplication (e.g., 6 • (2 • 3) = (6 • 2) • 3).

Grade 5

o Students will:

- 1. Use estimation skills to determine solutions to problems involving decimals.

- 2. Apply estimation skills to solve problems involving common percents and equivalent

- 3. Add and subtract decimal numbers with the same and different place values (e.g., 3.72 + 1.4) to solve problems.

- 4. Multiply and divide whole numbers and decimal numbers with 1- or 2-digit multipliers or divisors to solve problems.

- 5. Add and subtract fractions and mixed numbers to solve problems using a variety of methods (e.g., use fraction strips, find the least common denominator [LCD]).

Number Sense

Pre-Kindergarten

o Students will:

- 1. Demonstrate one-one correspondence in counting objects and matching groups of objects

- 2. Count in sequence to ten in the context of daily activities and play

- 3. Count objects in a set one-by-one from one through five

- 4. Identify and create sets of objects one through five

- 5. Identify and name numerals 0-10

Kindergarten

o Students will:

- 1. Compare a group or set to another group, set or numerical quantity and verbally explain which has more, less or equivalent quantities

- 2. Pair and count objects using one-one correspondence (e.g., one napkin for each child at snack time)

- 3. Count forward to twenty and backward from ten

- 4. Count objects in a set one-by-one from one through ten

- 5. Identify and create sets of objects zero through ten

- 6. Identify and write numerals zero through ten, in and out of sequence. Children may still be reversing some numerals

- 7. Identify the ordinal position (first through fifth) of objects

- 8. Combine and remove objects from sets and verbally describe the result (e.g., adding objects to a set makes the set larger, subtracting objects from a set makes the set smaller)

Grade 1

o Students will:

- 1. Use concrete models of tens and ones to develop the concept of place value.

- 2. Compare and order objects

- 3. Use ordinal numbers first through tenth to order objects.

- 4. Read and write numerals to 100.

- 5. Count as many as 100 objects by ones, twos, fives, and tens.

- 6. Investigate concepts of fractional parts (halves, thirds, fourths).

Grade 2

o Students will:

- 1. Use concrete models of hundreds, tens, and ones to develop the concepts of place value.

- 2. Demonstrate (using concrete objects, pictures, and numerical symbols) fractional parts including halves, thirds, and fourths.

- 3. Link place value concepts to the reading and writing of numbers (e.g., base-10 blocks).

- 4. Represent a number in a variety of ways (e.g., write the calendar day in different ways, write 15 as 8 + 7, write 25 as 2 tens + 5 ones or as 1 ten + 15 ones).

- 5. Write a number sentence to compare numbers less than 100 (e.g., 5 is more than 2, 3 is less than 7, page 51 comes after 50, and 51 is between 50 and 60).

- 6. Develop and use strategies of estimation (e.g., compose, decompose and regroup numbers, use knowledge of 10 to estimate quantities and sums [two numbers less than 10 can not add up to more than 20], use body parts to estimate measurements).

- 7. Determine whether a number is odd or even.

Grade 3

o   Students will:

- 1. Compare and order whole numbers up to 6 digits.

- 2. Compare and order fractions including halves, thirds fourths, eighths, and tenths using a model (e.g., fraction circles, pictures, egg cartons, fraction strips).

Grade 4

o   Students will:

- 1. Apply the concept of place value through 6 digits (e.g., write numbers in expanded form, play a trading game involving place value).

- 2. Read, write and rename whole numbers through 6 digits and decimal numbers to the hundredths (e.g., money, numerals to words).

- 3. Compare and order whole numbers and decimals to the hundredths place (e.g., pictures of shaded regions of two-dimensional figures, use >, < , = symbols).

- 4. Fractions

- 5. Use 0, 12 , and 1 or 0, 0.5, and 1, as benchmarks and place additional fractions and decimals on a number line (e.g., , , 0.7, 0.4).

- 6. Create physical and pictorial models of equivalent and nonequivalent fractional parts to be compared, added or subtracted (e.g., egg cartons, fraction strips, circles, and squares).

Grade 5

o   Students will:

- 1. Solve problems using decimal numbers to the 1000ths place.

- 2. Compare, convert, and order common fractions and decimals to the 100ths place to solve problems.

- 3. Represent with models the connection between fractions, decimals, and percents and be able to convert from one representation to another (e.g., use 10 x 10 grids, base-10 blocks; limit fractions to halves, thirds, fourths, fifths, tenths and twelths).

- 4. Explain verbally with manipulatives and diagrams 25%, 50%, 75%; use these percents to solve problems and relate them to their corresponding fractions and decimals.

- 5. Apply the basic properties of arithmetic: commutative, associative, distributive, and identity (e.g., show 2 (5 +1) = (2 • 5) + (2 • 1), given (5 + 1) +

(5 + 1) regroup to show this equals (5 + 5) + (1 + 1) concluding with (2 • 5) + (2 • 1)) to solve problems.

- 6. Identify and apply factors, multiples, prime, and composite numbers in a variety of problem-solving situations (e.g., build rectangular arrays for numbers 1-100 and classify as prime or composite).

Patterns/Algebra

Pre-Kindergarten

- o Students will:
  - 1. Sort and group objects into a set and explains verbally what the objects have in common (e.g. color, size, shape)
  - 2. Recognize patterns, can repeat them, and explain them verbally (red/black, red/black)

Kindergarten

- o Students will:
  - 1. Sort and group objects into a set and explain verbally what the objects have in common (e.g., color, size, shape)
  - 2. Explain verbally and extend simple patterns

Grade 1

- o Students will:
  - 1. Describe, extend and create a variety of patterns using concrete objects (e.g., sort a bag of objects by attributes and orally communicate the pattern for each grouping).
  - 2. Describe and extend number patterns in a variety of situations (e.g., addition charts, skip counting, calendars).

Grade 2

- o Students will:
  - 1. Describe, extend, and create patterns using symbols, shapes or designs (e.g., repeating and growing patterns made up of sets of shapes or designs, create patterns by combining different shapes and taking them apart).
  - 2. Formulate and record generalizations about number patterns in a variety of situations (e.g., addition and subtraction patterns, build a table showing the cost of one pencil at 10 cents, 2 pencils at 20 cents)

Grade 3

- o Students will:
    - 1. Describe (orally or in written form), create, extend and predict patterns using numbers (e.g., 3, 6, 9, 12…, use a function machine to generate input and output values for a table, show multiplication patterns on a hundreds chart).
    - 2. Analyze tables to formulate generalizations about patterns in a variety of situations (e.g., list the multiples of 5 in a table to show that multiples of 5 have a 0 or 5 in the ones place; given pairs of numbers with a common relationship, determine the rule and generate additional pairs with the same relationship).

Grade 4

- o Students will:
    - 1. Use a variety of techniques to generalize number patterns (e.g., use function machines and "t-tables" to demonstrate "What is the rule?").
    - 2. Solve simple open sentences involving operations on whole numbers (with a variable, e.g., a + 17 = 23).

Grade 5

- o Students will:
    - 1. Describe rules that produce patterns found in tables, graphs, and models, and use variables (e.g., boxes, letters, pawns, number cubes, or other symbols) to solve problems or to describe general rules in algebraic expression or equation form.
    - 2. Use algebraic problem-solving techniques (e.g., use a balance to model an equation and show how subtracting a number from one side requires subtracting the same amount from the other side) to solve problems.

Problem Solving

Grade 1

- o Students will:
    - 1. Use problem-solving approaches (e.g., act out situations, represent problems with drawings and lists, use concrete, pictorial, graphical, oral, written, and/or algebraic models, understand a problem, devise a plan, carry out the plan, look back).
    - 2. Formulate problems from everyday and mathematical situations (e.g., how many forks are needed?, how many students are absent?, how can we

share/divide these cookies?, how many different ways can we find to compare these fractions?).

- 3. Develop, test, and apply strategies to solve a variety of routine and nonroutine problems (e.g., look for patterns, make a table, process of elimination).

- 4. Verify and interpret results with respect to the original problem (e.g., students explain verbally why an answer makes sense, explain in a written format why an answer makes sense, verify the validity of each step taken to obtain a final result).

- 5. Distinguish between necessary and irrelevant information in solving problems (e.g., play games and discuss "best" clues, identify unnecessary information in written story problems).

Grade 2

o Students will:

- 1. Use problem-solving approaches (e.g., act out situations, represent problems with drawings and lists, use concrete, pictorial, graphical, oral, written, and/or algebraic models, understand a problem, devise a plan, carry out the plan, look back).

- 2. Formulate problems from everyday and mathematical situations (e.g., how many forks are needed?, how many students are absent?, how can we share/divide these cookies?, how many different ways can we find to compare these fractions?).

- 3. Develop, test, and apply strategies to solve a variety of routine and nonroutine problems (e.g., look for patterns, make a table, make a problem simpler, process of elimination, trial and error).

- 4. Verify and interpret results with respect to the original problem (e.g., students explain verbally why an answer makes sense, explain in a written format why an answer makes sense, verify the validity of each step taken to obtain a final result).

- 5. Distinguish between necessary and irrelevant information in solving problems (e.g., play games and discuss "best" clues, write riddles with sufficient information, identify unnecessary information in written story problems).

Grade 3

o Students will:

- 1. Use problem-solving approaches (e.g., act out situations, represent problems with drawings and lists, use concrete, pictorial, graphical, oral, written, and/or algebraic models, understand a problem, devise a plan, carry out the plan, look back).

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- 5. Distinguish between necessary and irrelevant information in solving problems (e.g., play games and discuss "best" clues, write riddles with sufficient information, identify unnecessary information in written story problems).

Grade 4

o Students will:

- 1. Use problem-solving approaches (e.g., act out situations, represent problems with drawings and lists, use concrete, pictorial, graphical, oral, written, and/or algebraic models, understand a problem, devise a plan, carry out the plan, look back).

- 2. Develop, test, and apply strategies to solve a variety of routine and nonroutine problems (e.g., look for patterns, make a table, make a problem simpler, process of elimination, trial and error).

- 3. Distinguish between necessary and irrelevant information in solving problems (e.g., play games and discuss "best" clues, write riddles with sufficient information, identify unnecessary information in written story problems).

Grade 5

o Students will:

- 1. Use problem-solving approaches (e.g., act out situations, represent problems with drawings and lists, use concrete, pictorial, graphical, oral, written, and/or algebraic models, understand a problem, devise a plan, carry out the plan, look back).

- 2. Formulate problems from everyday and mathematical situations (e.g., how many forks are needed?, how many students are absent?, how can we share/divide these cookies?, how many different ways can we find to compare these fractions?).

- 3. Develop, test, and apply strategies to solve a variety of routine and nonroutine problems (e.g., look for patterns, make a table, make a problem simpler, process of elimination, trial and error).

- 4. Verify and interpret results with respect to the original problem (e.g., students explain verbally why an answer makes sense, explain in a written

format why an answer makes sense, verify the validity of each step taken to obtain a final result).

- 5. Distinguish between necessary and irrelevant information in solving problems (e.g., play games and discuss "best" clues, write riddles with sufficient information, identify unnecessary information in written story problems).

Reasoning

Grade 1

- o Students will:
  - 1. Explain mathematical situations using patterns and relationships (e.g., identify patterns in situations, represent patterns in a variety of ways, extend patterns to connect with more general cases).
  - 2. Demonstrate thinking processes using a variety of age-appropriate materials and reasoning processes (e.g., manipulatives, models, known facts, properties and relationships, spatial, proportional, logical reasoning ["and" "or" "not"]

Grade 2

- o Students will:
  - 1. Explain mathematical situations using patterns and relationships (e.g., identify patterns in situations, represent patterns in a variety of ways, extend patterns to connect with more general cases).
  - 2. Demonstrate thinking processes using a variety of age-appropriate materials and reasoning processes (e.g., manipulatives, models, known facts, properties and relationships, inductive [specific to general], deductive [general to specific], spatial, proportional, logical reasoning ["and" "or" "not"] and recursive reasoning).
  - 3. Make predictions and draw conclusions about mathematical ideas and concepts. Predictions become conjectures and conclusions become more logical as students mature mathematically.

Grade 3

- o Students will:
  - 1. Explain mathematical situations using patterns and relationships (e.g., identify patterns in situations, represent patterns in a variety of ways, extend patterns to connect with more general cases).
  - 2. Make predictions and draw conclusions about mathematical ideas and concepts. Predictions become conjectures and conclusions become more logical as students mature mathematically.

Grade 4

- o Students will:
  - 1. Explain mathematical situations using patterns and relationships (e.g., identify patterns in situations, represent patterns in a variety of ways, extend patterns to connect with more general cases).

  - 2. Demonstrate thinking processes using a variety of age-appropriate materials and reasoning processes (e.g., manipulatives, models, known facts, properties and relationships, inductive [specific to general], deductive [general to specific], spatial, proportional, logical reasoning ["and" "or" "not"] and recursive reasoning).

  - 3. Make predictions and draw conclusions about mathematical ideas and concepts. Predictions become conjectures and conclusions become more logical as students mature mathematically.

Grade 5

- o Students will:
  - 1. Explain mathematical situations using patterns and relationships (e.g., identify patterns in situations, represent patterns in a variety of ways, extend patterns to connect with more general cases).

  - 2. Demonstrate thinking processes using a variety of age-appropriate materials and reasoning processes (e.g., manipulatives, models, known facts, properties and relationships, inductive [specific to general], deductive [general to specific], spatial, proportional, logical reasoning ["and" "or" "not"] and recursive reasoning).

  - 3. Make predictions and draw conclusions about mathematical ideas and concepts. Predictions become conjectures and conclusions become more logical as students mature mathematically.

Representation

Grade 1

- o Students will:
  - 1. Create and use a variety of representations appropriately and with flexibility to organize, record, and communicate mathematical ideas (e.g., dramatizations, manipulatives, drawings, diagrams, tables, graphs, symbolic representations).

  - 2. Use representations to model and interpret physical, social, and mathematical situations (e.g., counters, pictures, tally marks, number sentences, geometric models; translate between diagrams, tables, charts, graphs).

Grade 2

- o Students will:
  - 1. Create and use a variety of representations appropriately and with flexibility to organize, record, and communicate mathematical ideas (e.g., dramatizations, manipulatives, drawings, diagrams, tables, graphs, symbolic representations).
  - 2. Use representations to model and interpret physical, social, and mathematical situations (e.g., counters, pictures, tally marks, number sentences, geometric models;)

Grade 3

- o Students will:
  - 1. Create and use a variety of representations appropriately and with flexibility to organize, record, and communicate mathematical ideas (e.g., dramatizations, manipulatives, drawings, diagrams, tables, graphs, symbolic representations, number sentences).

Grade 4

- o Students will:
  - 1. Create and use a variety of representations appropriately and with flexibility to organize, record, and communicate mathematical ideas (e.g., dramatizations, manipulatives, drawings, diagrams, tables, graphs, symbolic representations).
  - 2. Use representations to model and interpret physical, social, and mathematical situations (e.g., counters, pictures, tally marks, number sentences, geometric models; translate between diagrams, tables, charts, graphs).

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- o Students will:
  - 1. Create and use a variety of representations appropriately and with flexibility to organize, record, and communicate mathematical ideas (e.g., dramatizations, manipulatives, drawings, diagrams, tables, graphs, symbolic representations).
  - 2. Use representations to model and interpret physical, social, and mathematical situations (e.g., counters, pictures, tally marks, number sentences, geometric models; translate between diagrams, tables, charts, graphs).

Shape and Space

Pre-Kindergarten

- o Students will:
  - 1. Sort and classify similar two dimensional objects found in the environment in play situations (2 squares, 2 stop signs)
  - 2. Identify and create circles, rectangles, squares, triangles and other common 2 dimensional shapes
  - 3. Build an increasing understanding of directionality, order and position of objects and words (e.g., on, under, above)

Kindergarten

- o Students will:
  - 1. Identify, compare and draw basic two-dimensional geometric shapes (e.g., circle, square, triangle, rectangle)
  - 2. Model and use words indicating relative position or direction (e.g., students describe the relationships between self and objects in space using on, above, below, beside, under on top of, behind and over)